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Effect of delay in thermal systems with long ducts

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Abstract

We analyze simple, one-dimensional models of thermal systems with long ducts in which there is a delay due to the time it takes the fluid to travel along the duct. A general solution of the dynamic problem of a single duct with time-dependent inlet and ambient temperatures is obtained, and several special cases are described in detail. Of particular interest is the periodic case in which the inlet and ambient temperatures are sinusoidal in time. Also presented is a model that includes the effect of thermal inertia of a heater located at the entrance to a duct for which the time-dependent temperature fields for periodic heating are calculated. Periodic behavior in closed loops and open loops with branching are also discussed.
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1. Introduction

In most thermal systems it is assumed that the future state is determined by the present. These systems are usually modeled by ordinary or partial differential equations. With this approach the modeling of networks of heating and cooling ducts with pumps, valves, heat exchangers and other components becomes very complicated. Some simplification is achieved by considering the advection of the temperature field so that the temperature at one point depends on the history of that at another. This leads to interesting dynamics that should be taken into account in designing a thermal control system. In this paper we will focus on ducts and networks, even though there are many other process control problems in which the sensors and actuators are separated and the signal from one and the needs of the other are shifted in time.

This work is intended as an introduction to duct-related thermal problems in which the dynamic behavior of a system is affected by built-in delays. A common feature of these problems is the finite time that a fluid takes to traverse the length of a duct. The present goal is to determine the kind of physical effects that delay can cause, the ultimate goal being an understanding of thermal networks and the design of proper control strategies. Delay, of course, is of significance only in the dynamics of time-dependent systems. We will assume, however, that the flow is generated at a constant rate by a pump or similar device, and consider only the thermal aspects of the problem.

2. Delay equations and their applications

There are systems whose behavior depends significantly on past events or on some other function of the present state. These are modeled by *functional equations* in which the unknown function occurs with different arguments. An extensive literature on functional equations exists (see, for example, [1,2]); some examples are given in the upper half of Table 1. A special case is that of *difference equations* [3], in which the unknown function is evaluated at arguments of the form $(t + constant)$. The equation may be algebraic or differential. Differential–difference equations can be classified, as shown in the lower half of Table 1, based on the sign of the constant. We are interested here in *delay equations* [4,5] which occur frequently in the analysis of thermal systems. In these the equation expresses some derivative of the unknown function evaluated at one instant in terms of its lower order derivatives, if any, at the same or earlier instants.

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Nomenclature

A	area	m^2
c	specific heat	$J\cdot kg^{-1}\cdot K^{-1}$
i	current	A
I	nondimensional current	
L	duct length	m
M	heater mass	kg
$\dot{m}$	mass flow rate	$kg\cdot s^{-1}$
p	mass flow fraction	
P	perimeter of cross-section	m
q	ratio of residence times	
Q	heat rate	W
R	electrical resistance	Ω
t	time	s
T	temperature	°C
T_h	temperature of fluid after heater	°C
T_H	temperature of heater	°C
ΔT	characteristic temperature difference	°C
U	heat transfer coefficient	$W\cdot m^{-2}\cdot K^{-1}$
x	axial coordinate	m

Greek symbols

α'	convection parameter	
α''	heat capacity parameter	
γ	heat loss parameter	
ρ	fluid density	$kg\cdot m^{-3}$
τ	residence time	s
ϕ	phase angle	
ω	frequency of $T_{in}(t)$	s^{-1}
Ω	frequency of $T_\infty(t)$	s^{-1}

Subscripts

H	heater
in	inlet
out	outlet
0	initial condition
∞	ambient
$\widehat{(\cdot)}$	amplitude
$\overline{(\cdot)}$	mean value
$(\cdot)^*$	dimensional quantity

Table 1
Functional equations and differential–difference equations, t_0, t_1 and $t_2 > 0$

Type of equation	Example
Algebraic	$x(2t) = 0.5[x(t) + t]$
Ordinary differential	$\ddot{x}(t) = \dot{x}(t/3) - x(t-2)$
Partial differential	$\partial u/\partial t = \partial u/\partial x + u(x, t - t_0)$
Integral	$x(t) = \int_t^{t+1} k(s)x(s)\,ds$
Integro-differential	$\dot{x}(t) = -\int_{t-r}^{t} a(t-u)g(x(u))\,du$
Retarded (delay)	$\ddot{x}(t) = \dot{x}(t - t_1) - x(t - t_2)$
Advanced	$\dot{y}(t) = y(t + t_0)$
Neutral	$\dot{x}(t) - C\dot{x}(t - t_0) - Dx(t - t_0) = 0$
Mixed	$\dot{x}(t) + Ax(t - t_0) + Bx(t + t_0) = 0$

There are mathematical aspects of delay equations, such as stability and chaotic behavior [6], that are of interest in practical applications. Delay equations have been used in a variety of different fields, such as biomedical engineering [7,8] and economics [9]. However, the literature contains few applications to thermal systems. One exception is in the area of heat exchangers that have been studied by Górecki et al. [5] and Huang et al. [10], among others. Zhang and Nelson [11] also modeled the effect of a variable-air-volume ventilating system on a building using delay, and Saman and Mahdi [12] analyzed pipe and fluid temperature variations due to flow.

3. Model of heat transfer in a duct

We begin with a model of thermal effects in a constant-area duct with a constant flow rate that is schematically shown in Fig. 1. The inlet temperature is $T_{in}^*(t^*)$, the mass flow rate of fluid is $\dot{m}$, and the outlet temperature is $T_{out}^*(t^*)$. The duct is subject to heat loss through its surface of the form $UP[T^* - T_\infty^*(t^*)]$ per unit length, where the local fluid temperature is $T^*(x^*, t^*)$ and the ambient temperature is $T_\infty^*(t^*)$. U is the overall heat transfer coefficient that is assumed constant, and P is the perimeter at a cross section of the duct. x^* is the longitude coordinate measured from the inlet and t^* is time.

We assume that the flow is one-dimensional, and neglect axial conduction through the fluid and the duct. The governing energy balance per unit length gives

$$\dot{m}c\frac{\partial T^*}{\partial x^*} + \rho Ac\frac{\partial T^*}{\partial t^*} + UP[T^* - T_\infty^*(t^*)] = 0 \tag{1}$$

where A is the cross-sectional area of the duct and c is the specific heat at constant pressure of the fluid. The ambient temperature can be written as $T_\infty^*(t^*) = \overline{T}_\infty^* + T_\infty^{'*}(t^*)$, where the time-averaged and fluctuating parts have been separated. We use the nondimensional space, time and temperature variables

$$x = \frac{x^*}{L}, \qquad t = \frac{t^*}{\tau}, \qquad T = \frac{T^* - \overline{T}_\infty^*}{\Delta T} \tag{2}$$

where L is the length of the duct, $\tau = A\rho L/\dot{m}$ is the time taken to traverse the length of the duct, i.e., the residence time. In this section the characteristic temperature difference ΔT is arbitrary. Thus, Eq. (1) becomes

$$\frac{\partial T}{\partial x} + \frac{\partial T}{\partial t} + \gamma[T - T_\infty'(t)] = 0 \tag{3}$$

where $T_\infty'(t) = T_\infty^{'*}(t^*)/\Delta T$. The parameter $\gamma = UPL/\dot{m}c$ represents the heat loss to the surroundings.

$$T_\infty^*(t^*)$$

$$T_{in}^*(t^*) \qquad \dot{m} \longrightarrow \qquad T(x^*,t^*) \qquad\qquad T_{out}^*(t^*)$$
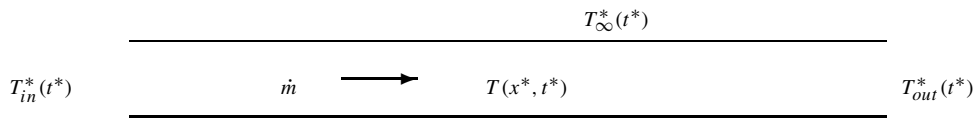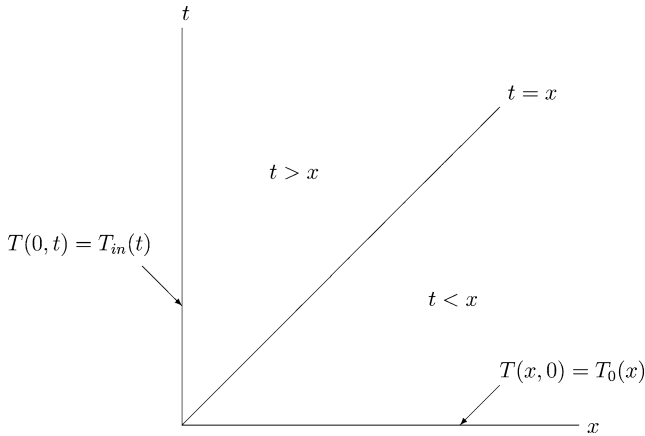
Fig. 1. Flow in duct with heat loss.

Fig. 2. Boundary and initial conditions in x–t space.

Using the method of characteristics [13], the first-order partial differential equation (3) is reduced to

$$\mathrm{d}x = \mathrm{d}t = -\frac{\mathrm{d}T}{\gamma[T - T_\infty'(t)]} \tag{4}$$

From this, two integrals

$$C_1 = x - t, \tag{5}$$

$$C_2 = e^{\gamma t} T - \gamma \int_0^t e^{\gamma s} T_\infty'(s)\,\mathrm{d}s \tag{6}$$

are obtained, and the general solution can be written as one as a function of the other. Thus, for example, we can write

$$T(x,t) = e^{-\gamma t}\left[f(x-t) + \gamma \int_0^t e^{\gamma s} T_\infty'(s)\,\mathrm{d}s \right] \tag{7}$$

where f is an arbitrary function. The boundary conditions $T(0,t) = T_{in}(t)$ and $T(x,0) = T_0(x)$ are shown in Fig. 2. Using these, the solution becomes

$$T(x,t) = \begin{cases} T_{in}(t-x)e^{-\gamma x} + \gamma e^{-\gamma t} \int_{t-x}^t e^{\gamma s} T_\infty'(s)\,\mathrm{d}s \\ \quad \text{for } t \geqslant x \\ T_0(x-t)e^{-\gamma t} + \gamma e^{-\gamma t} \int_0^t e^{\gamma s} T_\infty'(s)\,\mathrm{d}s \\ \quad \text{for } t < x \end{cases} \tag{8}$$

The $t < x$ part of the solution is applicable to the brief, transient period of time in which the fluid at time $t = 0$ has still not left the duct. The later $t > x$ part depends on the temperature of the fluid entering at $x = 0$. The temperature, $T_{out}(t)$, at the outlet section, $x = 1$, is given by

$$T_{out}(t) = \begin{cases} T_{in}(t-1)e^{-\gamma} + \gamma e^{-\gamma t} \int_{t-1}^t e^{\gamma s} T_\infty'(s)\,\mathrm{d}s \\ \quad \text{for } t \geqslant 1 \\ T_0(1-t)e^{-\gamma t} + \gamma e^{-\gamma t} \int_0^t e^{\gamma s} T_\infty'(s)\,\mathrm{d}s \\ \quad \text{for } t < 1 \end{cases} \tag{9}$$

It can be observed that, after an initial transient, the inlet and outlet temperatures are related by a unit delay. The outlet temperature is also affected by the heat loss parameter, γ, and the ambient temperature change, $T_\infty'(t)$. The following are some special cases of the outlet temperature.

3.1. Perfectly insulated duct

If $\gamma = 0$ the outlet temperature simplifies to

$$T_{out}(t) = \begin{cases} T_{in}(t-1) & \text{for } t \geqslant 1 \\ T_0(1-t) & \text{for } t < 1 \end{cases} \tag{10}$$

After the initial transient, the outlet temperature is the same as the inlet temperature but at a previous instant in time.

3.2. Constant ambient temperatures

For this $T_\infty' = 0$, and Eq. (9) becomes

$$T_{out}(t) = \begin{cases} T_{in}(t-1)e^{-\gamma} & \text{for } t \geqslant 1 \\ T_0(1-t)e^{-\gamma t} & \text{for } t < 1 \end{cases} \tag{11}$$

This is similar to Eq. (10), but with an exponential drop due to heat transfer.

3.3. Periodic inlet and ambient temperature

We take

$$T_{in}(t) = \overline{T}_{in} + \widehat{T}_{in} \sin \omega t \tag{12}$$

$$T_\infty'(t) = \widehat{T}_\infty \sin \Omega t \tag{13}$$

so that Eq. (9) becomes

$$T_{out}(t) = \begin{cases} [\overline{T}_{in} + \widehat{T}_{in} \sin \omega(t-1)]e^{-\gamma} \\ \quad + \widehat{T}_\infty \gamma \sqrt{\frac{-2e^{-\gamma}\cos 1 + e^{-2\gamma}}{\gamma^2 + \Omega^2}} \sin(\Omega t + \phi) \\ \quad \text{for } t \geqslant 1 \\ T_0(1-t)e^{-\gamma t} \\ \quad + \frac{\gamma}{\gamma^2 + \Omega^2}\widehat{T}_\infty \sqrt{\gamma^2 + \Omega^2} \sin(\Omega t + \phi') \\ \quad \text{for } t < 1 \end{cases} \tag{14}$$

where

Heater $T_\infty(t)$

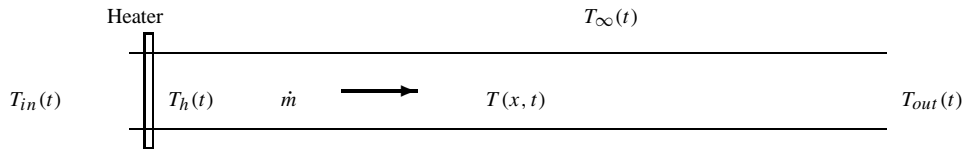$T_{in}(t)$ $T_h(t)$ $\dot{m}$ → $T(x,t)$ $T_{out}(t)$

Fig. 3. Flow in duct with heater.

$$\tan\phi = -\frac{\gamma(1 - e^{-\gamma}\cos 1) + e^{-\gamma}\Omega\sin 1}{\Omega(1 - e^{-\gamma}\cos 1) - \gamma e^{-\gamma}\sin 1} \tag{15}$$

$$\tan\phi' = -\frac{\Omega}{\gamma} \tag{16}$$

The outlet temperature has frequencies that come from oscillations in the inlet as well as the ambient temperatures. A properly-designed control system that senses the outlet temperature must take the frequency dependence of its amplitude and phase into account. There are several complexities that must be considered in practical applications to heating or cooling networks, some of which are analyzed below.

4. Thermal inertia and periodic heating

In the previous section, the inlet temperature $T_{in}(t)$ could be varied at will. In reality though, heating or cooling is usually through devices that have a finite response time. To study this let us consider an electric heater placed at the inlet of the duct, as shown in Fig. 3. The current in the heater can be either i or zero. The finite mass of the heater, M, makes it a first-order system. The heater raises the temperature of the fluid from $T_{in}(t)$ to $T_h(t)$, which is the temperature just after the heater, while $T_H(t)$ is the temperature of the heater itself. The nondimensional governing equations for heat transfer from the heater to the fluid and for the temperature of the heater are

$$T_h - T_{in} = \alpha'\left[T_H - \frac{1}{2}(T_{in} + T_h)\right] \tag{17}$$

$$\alpha''\frac{dT_H}{dt} = \alpha'\left[\frac{1}{2}(T_{in} + T_h) - T_H\right] + I(t)^2 \tag{18}$$

where $\alpha' = h_H A_H/\dot{m}c$ and $\alpha'' = Mc_H/\dot{m}c\tau$. h_H is the heat transfer coefficient between the heater and the fluid, A_H is the heat transfer surface area, and c_H is the specific heat of the material of the heater. The average fluid temperature $(T_{in} + T_h)/2$ is used for the heat transfer calculation. The current $I(t)$ is nondimensionalized by $(\dot{m}c\Delta T/R)^{1/2}$, where R is the electrical resistance of the heater. Choosing $\Delta T = i^2 R/\dot{m}c$, we have $I(t) = 1$ for heater on, and $I(t) = 0$ for heater off.

Eliminating T_H from Eqs. (17) and (18), we get

$$\alpha\frac{dT_h}{dt} + T_h = T_{in}(t) + \beta\frac{dT_{in}}{dt} + I(t)^2 \tag{19}$$

where $\alpha = \alpha''(1/\alpha' + 1/2)$ and $\beta = \alpha''(1/\alpha' - 1/2)$. Solving, we have

$$T_h(t) = T_h(0)e^{-t/\alpha} + \frac{\beta}{\alpha}\left[T_{in}(t) - e^{-t/\alpha}T_{in}(0)\right]$$
$$+ \frac{e^{-t/\alpha}}{\alpha}\int_0^t e^{s/\alpha}\left[\frac{\alpha - \beta}{\alpha}T_{in}(s) + I(s)^2\right]ds \tag{20}$$

$T_h(t)$ is the entrance temperature to be used instead of $T_{in}(t)$ in Eqs. (8) and (9) to obtain the temperature distribution in the duct.

When $T_\infty' = 0$ and T_{in} is constant we find that

$$T_{out}(t) = \begin{cases} [T_0(0)e^{-(t-1)/\alpha} + T_{in}(1 - e^{-(t-1)/\alpha}) \\ \quad + \frac{e^{-(t-1)/\alpha}}{\alpha}\int_0^{t-1} e^{s/\alpha}I(s)^2\,ds]e^{-\gamma} \\ \quad \text{for } t \geqslant 1 \\ T_0(1-t)e^{-\gamma t} \quad \text{for } t < 1 \end{cases} \tag{21}$$

We will now assume that both the heating and the response of the system are periodic in time. At the instant the heater switches on, the temperature distribution is $T_1(x)$. The heater runs for a time interval of t_1 with $I(t) = 1$ at which time it switches off. The temperature distribution at that instant is $T_2(x)$. The heater-off interval with $I(t) = 0$ is t_2, at the end of which the temperature distribution goes back to $T_1(x)$, and the heater switches on. Being periodic, the temperature distributions should satisfy

$$T_2(x) = \begin{cases} [T_{in} + 1 + e^{-(t_1-x)/\alpha}\{T_1(0) - T_{in} - 1\}]e^{-\gamma x} \\ \quad \text{if } x \leqslant t_1 \\ T_1(x - t_1)e^{-\gamma t_1} \quad \text{if } x > t_1 \end{cases} \tag{22}$$

and

$$T_1(x) = \begin{cases} [T_{in} + e^{-(t_2-x)/\alpha}\{T_2(0) - T_{in}\}]e^{-\gamma x} \\ \quad \text{if } x \leqslant t_2 \\ T_2(x - t_2)e^{-\gamma t_2} \quad \text{if } x > t_2 \end{cases} \tag{23}$$

Fig. 4(a) shows the spatial temperature distributions at the instant the heater is started and when it is stopped. In Fig. 4(b) the temporal behaviors of the temperature of the fluid just after the heater, $x = 0$, and at the outlet, $x = 1$, are indicated; $t = 0$ corresponds to the instant the heater is switched on. In this example it appears that during the interval the heater is on, $0 < t \leqslant 0.25$, the temperature of the fluid at the outlet is actually decreasing. Thus there are obvious difficulties associated with using the temperature of the outlet directly as a control signal to switch the heater off and on.

5. Closed loop

In a closed loop the effect of delay at the outlet is immediately fed back into the inlet. Let us consider the loop
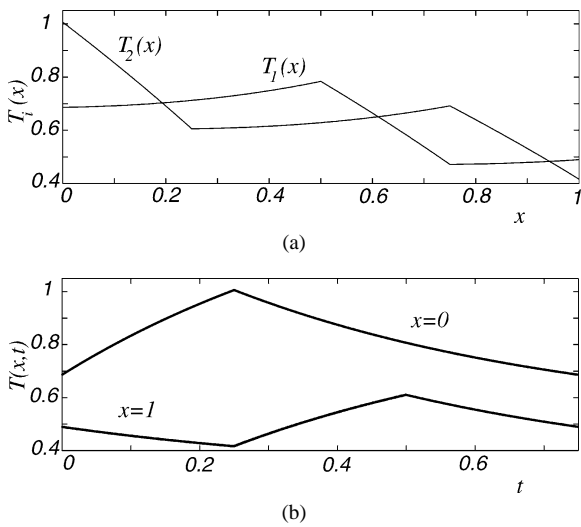
Fig. 4. Temperatures for duct with periodic heating, (a) spatial distribution, $T_1(x)$ is at end of off-period and $T_2(x)$ is at end of on-period, and (b) temporal dependence, $x = 0$ is just after heater and $x = 1$ is at end of duct. Parameters: $t_1 = 0.25$, $t_2 = 0.5$, $\alpha = 0.5$, $T_{\text{in}} = 0.5$, $\gamma = 0.5$.
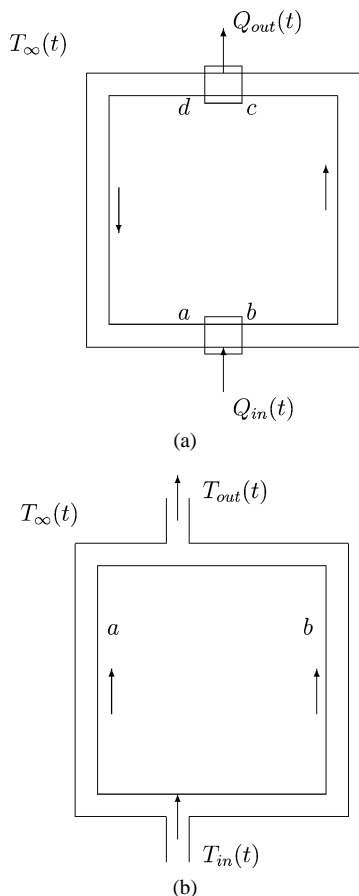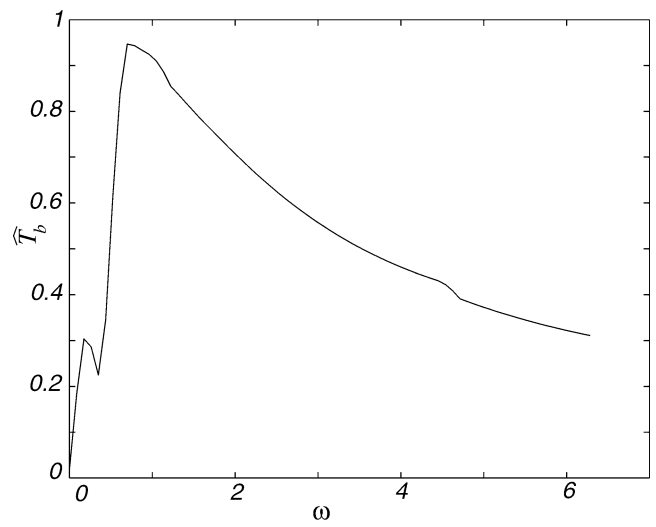

Fig. 6. Amplitude of temperature at point b, $\widehat{T}_b$, for a closed loop. Parameters: $\gamma_{bc} = \gamma_{da} = 0.01$, $\overline{Q}_{\text{in}} = \overline{Q}_{\text{out}} = 0$, $\widehat{Q}_{\text{in}} = \widehat{Q}_{\text{out}} = 1$, $\phi = 0$, $q = 1$.

between a and b by a heat exchanger, and taken out between c and d by another.

Using Eq. (9), we can write relations between the inlet and outlet of sides bc and da of the loop to get

$$T_c(t) = \left[T_b(t-1)e^{-\gamma_{bc}} + \gamma_{bc}e^{-\gamma_{bc}t} \int_{t-1}^{t} e^{\gamma_{bc}s} T_\infty'(s)\,\mathrm{d}s \right] \tag{24}$$

$$T_a(t) = \left[T_d\left(t - \frac{1}{q}\right)e^{-\gamma_{da}} \right.$$
$$\left. + q\gamma_{da}e^{-q\gamma_{da}t} \int_{qt-1}^{qt} e^{q\gamma_{da}s} T_\infty'(s)\,\mathrm{d}s \right] \tag{25}$$

where $T_a(t)$, $T_b(t)$, $T_c(t)$ and $T_d(t)$ are the temperatures at the respective points, $T_0(x)$ is the initial distribution of temperature, and $q = \tau_{bc}/\tau_{da}$. We have ignored the initial transient. Energy balances at the heat exchangers give

$$T_b(t) - T_a(t) = Q_{\text{in}}(t) \tag{26}$$
$$T_c(t) - T_d(t) = Q_{\text{out}}(t) \tag{27}$$

where the heat rates have been nondimensionalized by $\dot{m}c\Delta T$. Fig. 6 shows the amplitude response of the loop to a periodic heat flux where

$$Q_{\text{in}}(t) = \overline{Q}_{\text{in}} + \widehat{Q}_{\text{in}} \sin \omega t \tag{28}$$
$$Q_{\text{out}}(t) = \overline{Q}_{\text{out}} + \widehat{Q}_{\text{out}} \sin(\omega t + \phi) \tag{29}$$

The amplitude is greatest for $\omega = 1$.

6. Branching

Finally, we consider the branched open loop shown schematically in Fig. 5(b). The two branches a and b
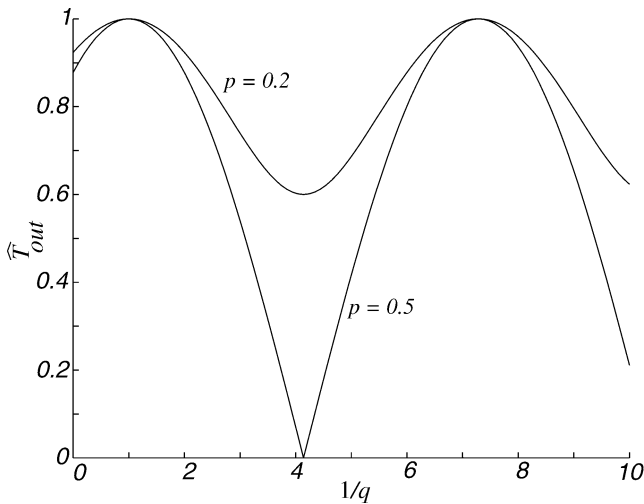

Fig. 5. (a) Closed loop, (b) open loop with branching.

schematically shown in Fig. 5(a). The flow is pumped in a counterclockwise direction at a constant mass flow rate $\dot{m}$. a, b, c and d are four points along the loop. Heat is put in

Fig. 7. Amplitude of outlet temperature, $\widehat{T}_{out}$ for branching. Parameters: $\overline{T}_{in} = 0$, $\widehat{T}_{in} = 1$, $\omega = 1$, $\gamma_a = \gamma_b = 0$. Results for two different values of p are shown.

have different geometrical, flow and thermal characteristics. Because of this the delay caused in each branch as well as the heat loss parameters are different. The temperature at the outlet is a result of the mixing of the two streams.

Nondimensionalizing the governing equations with respect to the characteristic values in branch a, we find, on ignoring the initial transient, that

$$
T_{out}(t) = p \left[T_{in}(t-1)e^{-\gamma_a} + \gamma_a e^{-\gamma_a t} \int_{t-1}^{t} e^{\gamma_a s} T'_{\infty,a}(s)\, ds \right]
$$
$$
+ (1-p) \left[T_{in}\left(t - \frac{1}{q}\right)e^{-\gamma_b} \right.
$$
$$
\left. + q\gamma_b e^{-q\gamma_b t} \int_{qt-1}^{qt} e^{q\gamma_b s} T'_{\infty,b}(s)\, ds \right] \tag{30}
$$

where the mass flow fraction is

$$
p = \frac{\dot{m}_a}{\dot{m}_a + \dot{m}_b} \tag{31}
$$

and the ratio of residence times is

$$
q = \frac{\tau_a}{\tau_b} \tag{32}
$$

The parameters in each branch are taken to be different.

As an example we consider a sinusoidal inlet temperature where T_{in} is given by Eq. (12). For simplicity we take $\gamma_a = \gamma_b = 0$. The outlet temperature, $T_{out}(t)$, is also sinusoidal, but the interference between the flows which come through the two different branches affect its amplitude and phase. Fig. 7 shows the amplitude $\widehat{T}_{out}$ of $T_{out}(t)$ for different values of p. In the extreme case of $p = 0.5$, $q = 0.2414$, the two branches cancel each other completely. It can be readily appreciated that using the outlet temperature for control purposes in this case would be impossible.

7. Conclusions

Delay in thermal systems with long ducts is due to the finite time that the fluid takes to traverse its length. This delay time becomes an important factor in dynamic problems such as when the inlet or ambient temperature is time-dependent, or when a sensor at one point controls an actuator at another. Delay must be considered when modeling the system since it can adversely affect its performance, or even contribute to the instability of a thermal control system.

We have discussed the behavior of some ducted systems along with the effects of thermal inertia, closed loops and branching. The ultimate objective is to understand and design better heating and cooling networks for buildings and districts which are often very complicated.

Acknowledgements

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